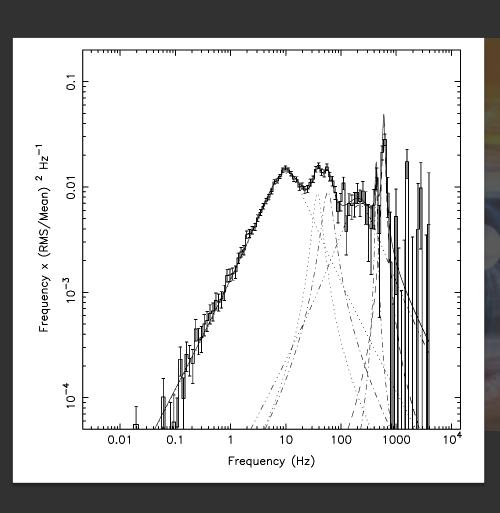
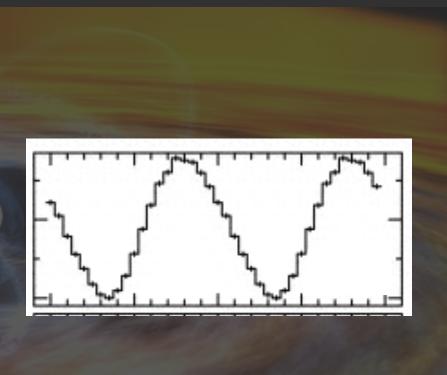
Quasi-periodic modulation of accreting millisecond pulsars

QPO/Pulse coupling



QPO/Pulse coupling

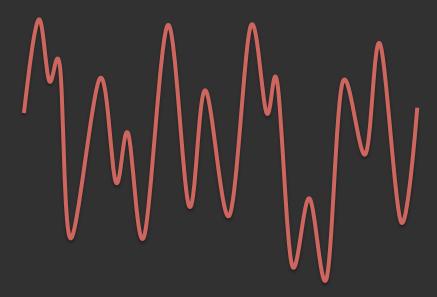




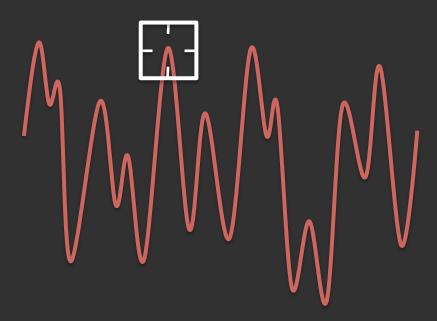
QPO/Pulse coupling

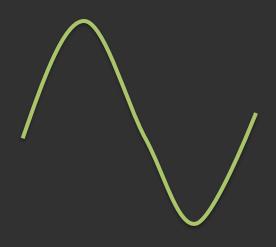
- Tells us something about the pulsar emission
 - Hotspot shape
 - Beaming angle
- Tells us something about the aperiodic emission
 - Accretion geometry
- Complementary to spectral timing type studies

Consider a light curve with a prominent QPO

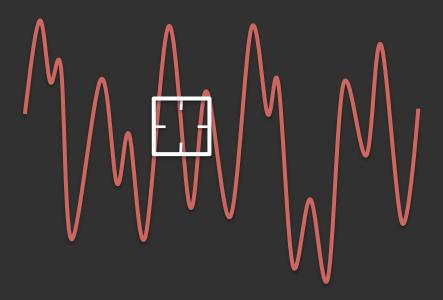


Fold a pulse profile for potions of the light curve



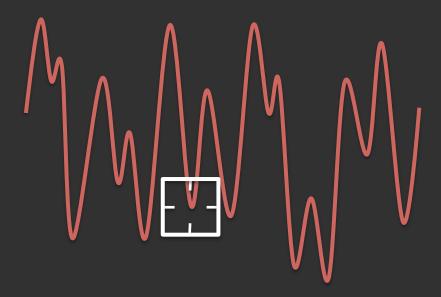


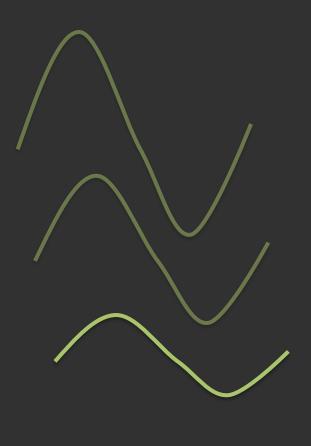
See how the pulse changes with QPO phase





See how the pulse changes with QPO phase



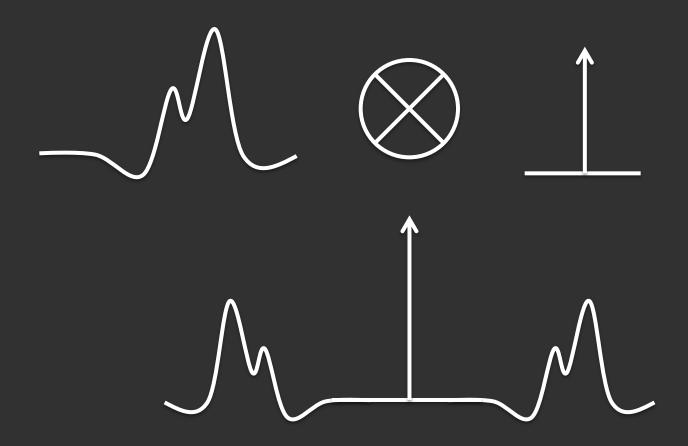


- Heuristic
- Time domain selection needed
- Qualitative
 - statistics not well defined
 - not clear what fit parameters mean

a better way?

- All information exists in the Fourier Domain
- Multiplied signals are Fourier Domain convolutions
- Convolution with a pulse spike is easy!

pulsar sidebands



FFT Sampling distorts the profile

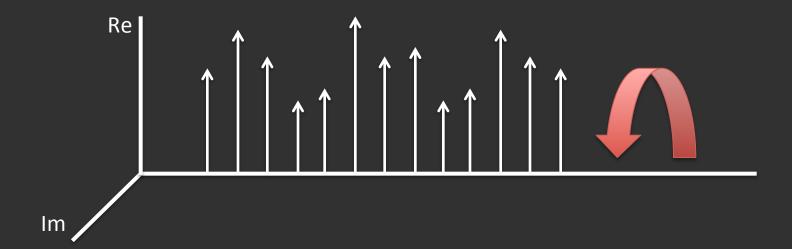


FFT Sampling distorts the profile

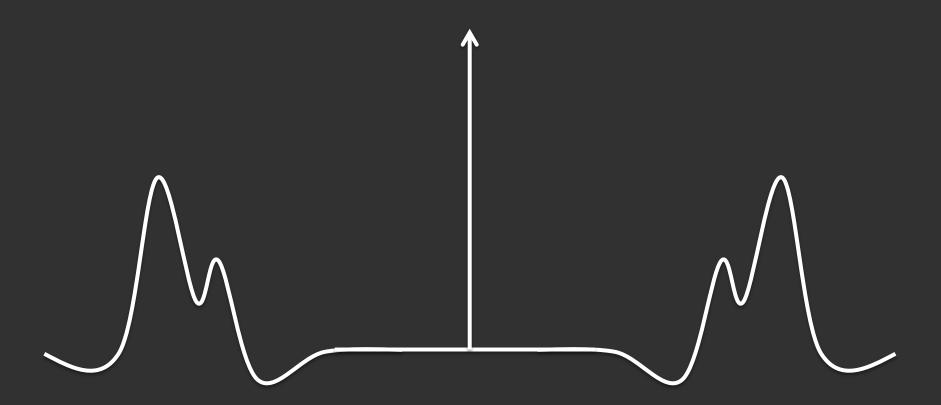
- Spectral leakage
 - Broadening of the powers
 - Bias in the phase

correcting the distortion

- Take the light curve
- Rotate into the complex domain
- Then FFT

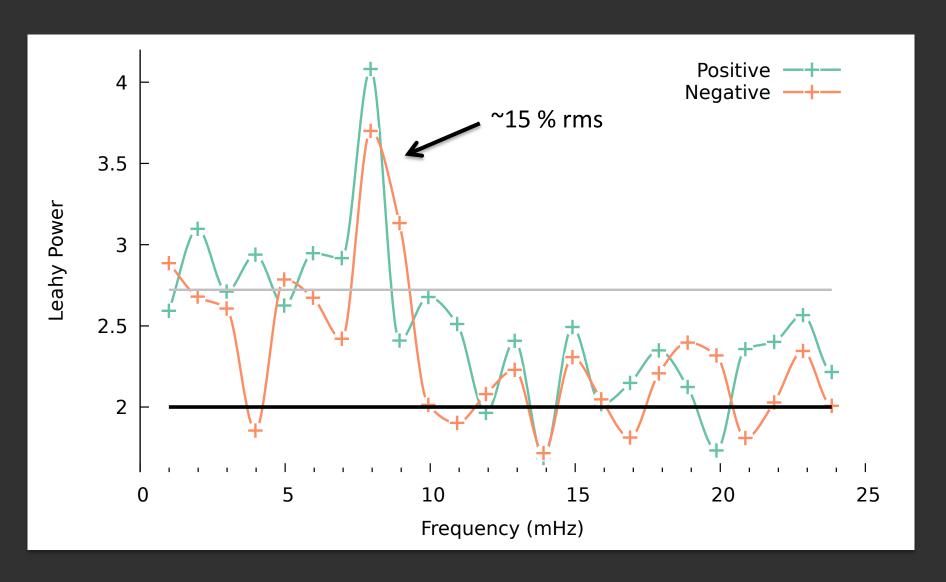


pulse amplitude modulation



pulse amplitude modulation

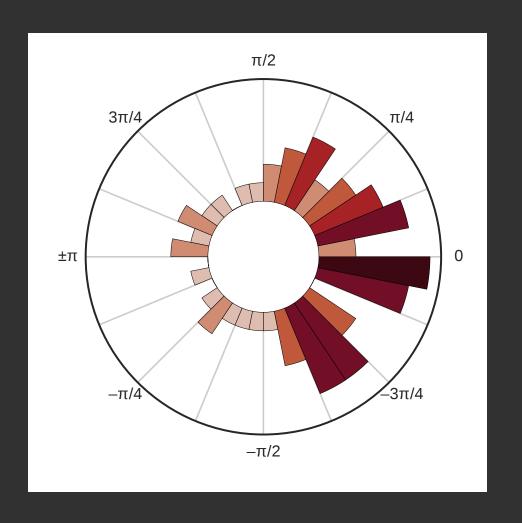
- Now the sidebands are properly sampled
- The two bands are complex conjugates
- We can calculate the:
 - Coupling power spectrum
 - Coupling coherence measure
 - Coupling phase lags



Phase distribution between sidebands

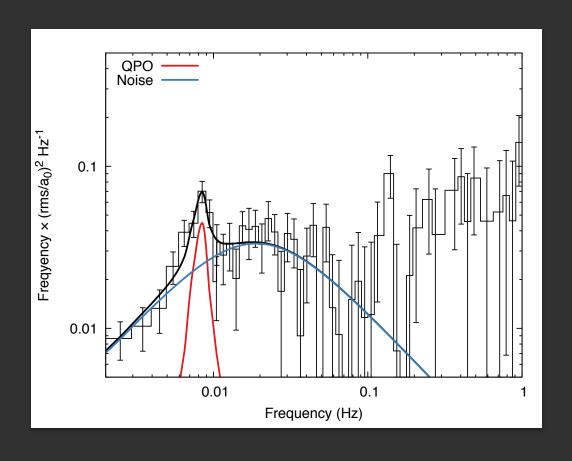
$$S_{+}(f) S_{-}(f)$$

= $A_{+}A_{-} e^{2\pi i (\Phi_{+} + \Phi_{-})}$
= $A^{2} e^{2\pi i (\Phi_{-} + \Phi_{-})}$



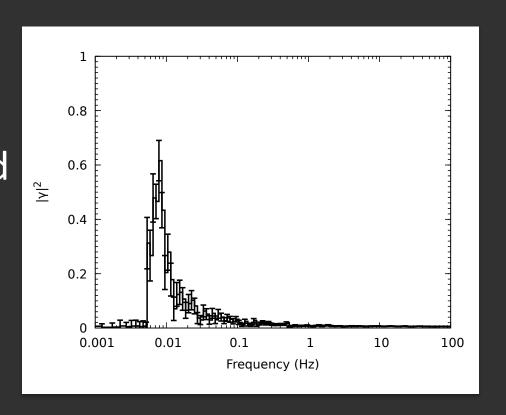
A coupling pds.

Both the QPO and the noise are modulating the pulsations.



Coupling coherence.

Knowing the sideband complex Fourier samples, we can now calculate cross coherence measures

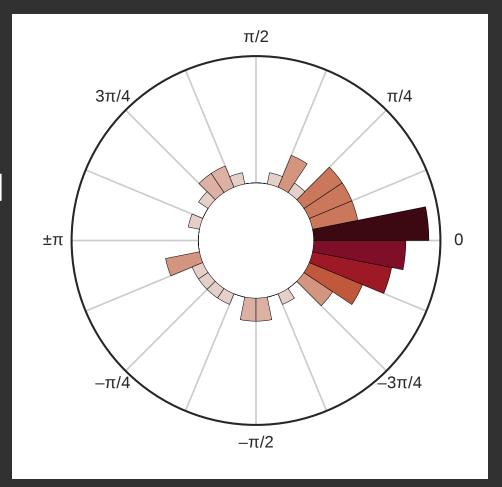


Coupling phase lag

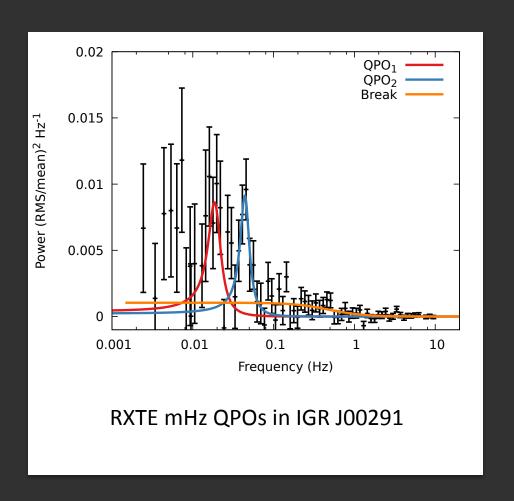
Phase distribution between sideband and the direct low frequency spectrum

$$S_{QPO}(f) S_{Pulse}(f)^*$$

= $A_Q A_P e^{2\pi i (\Phi_Q - \Phi_P)}$



Works even for weak QPOs that are not visible in the the light curve directly.



high throughput x-ray timing

- The obvious improvements apply
 - Higher count rates means faster convergence
 - Less phase smearing
 - Better energy resolution (higher s/n)
 - Energy spectra for QPO, Pulse AND coupling separately

Summary

- A Fourier domain framework for QPO/Pulse coupling studies
- Meaningful measures of coupling and interaction
- Well defined statistics